

## Exercise 4

Describe the geometric meaning of the following mappings in cylindrical coordinates:

(a)  $(r, \theta, z) \mapsto (r, \theta, -z)$

(b)  $(r, \theta, z) \mapsto (r, \theta + \pi, -z)$

(c)  $(r, \theta, z) \mapsto (-r, \theta - \pi/4, z)$

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### Solution

#### Part (a)

This mapping reflects the point across the  $xy$ -plane.

#### Part (b)

This mapping rotates the point  $180^\circ$  around the  $z$ -axis (as indicated by the right-hand corkscrew rule) and then reflects it across the  $xy$ -plane. In other words, it reflects the point across the origin.

#### Part (c)

This mapping rotates the point  $135^\circ$  around the  $z$ -axis (as indicated by the right-hand corkscrew rule). Note that changing  $r$  to  $-r$  is effectively the same as adding  $\pi$  to  $\theta$ . In total, the change in  $\theta$  is  $\pi - \pi/4 = 3\pi/4 = 135^\circ$ .